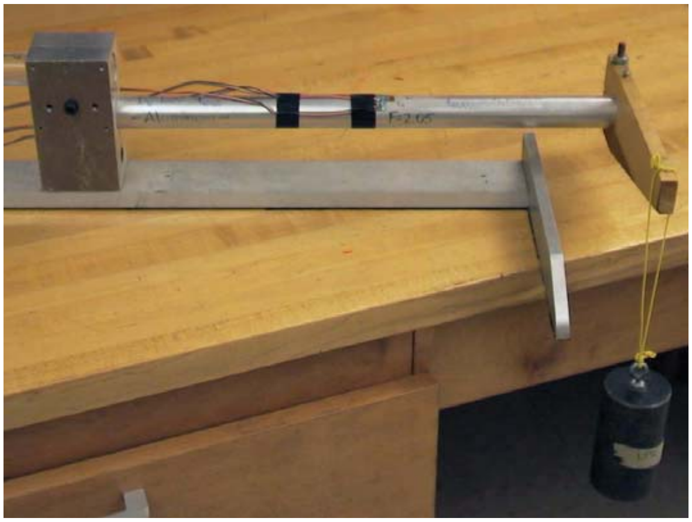
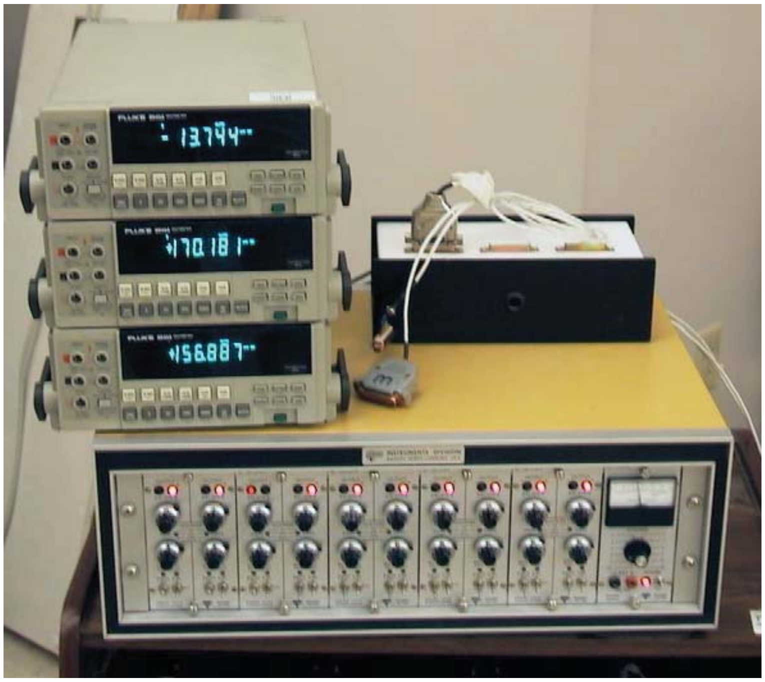
Laboratory 13

<u>Strain Gages</u>

Required Special Equipment:

- strain gage conditioner and amplifier system (Measurements Group 2120A and 2110A modules)
- strain gage interface box and cabling
- custom-made apparatus containing an aluminum tube with a strain gage Rosette mounted on its top surface.





13.1 Introduction

The intent of this laboratory exercise is to familiarize the student with the use and application of strain gages. In particular, this exercise will utilize a rectangular strain gage rosette, strain gage conditioner and a voltmeter for the determination of strains within a loaded specimen. A foil strain gage and a rectangular strain gage rosette are illustrated below.

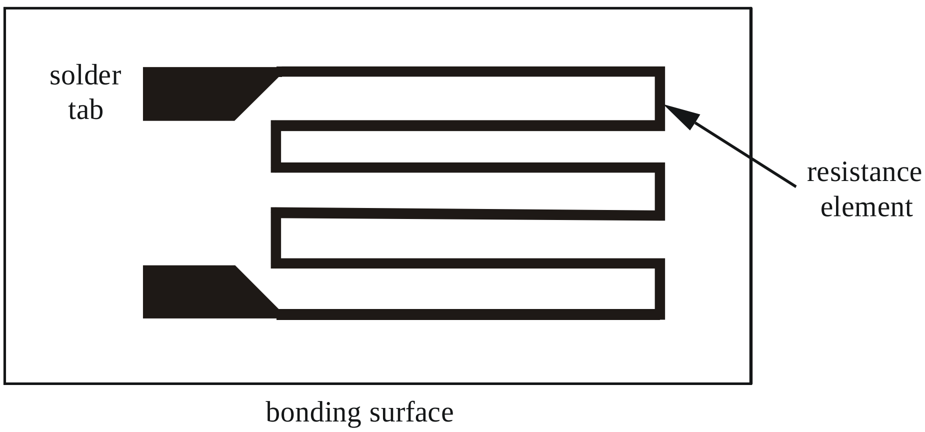


Figure 13.1 Foil Gage

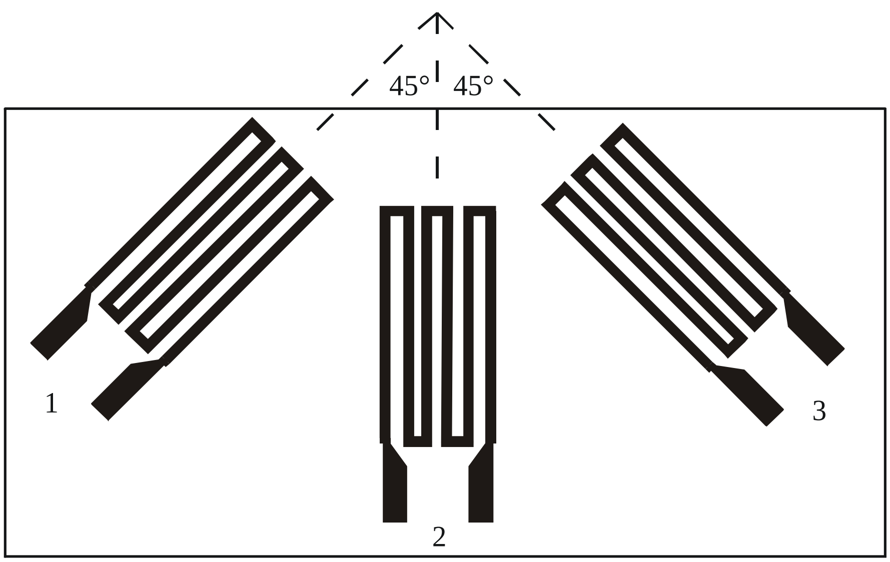


Figure 13.2 Rectangular Strain Gage Rosette

13.2 Theory

The basic principle under which the strain gage operates is the fact that the electrical resistance of a conductor changes in response to a mechanical deformation:

$$R = \rho\frac{L}{A} \tag{13.1}$$

where

R = resistance of conductor

ρ = resistivity of material

L = length of conductor

A = cross-sectional area of conductor

Relating the above definition to Poisson's ratio and strain yields the following:

$$F = 1 + 2\nu + \frac{1}{\varepsilon}\frac{\Delta\rho}{\rho} \tag{13.2}$$

$$\varepsilon = \frac{1}{F}\frac{\Delta R}{R} \tag{13.3}$$

where

F = gage factor

ν = Poisson's ratio

ε = axial strain

ΔR = change in gage resistance due to deformation

R = undeformed gage resistance

The strain gage conditioner consists of several channels, each containing a bridge/amplifier circuit. Each channel outputs a bridge detector potential, V_o, that is related to the strain in the gage connected to that channel. A bridge circuit is illustrated in Figure 13.3. For a balanced bridge (i.e., $V_o = 0$) the condition $R_1R_3 = R_2R_4$ must be satisfied. Thus, once the bridge is balanced for a no strain condition, a strain induced on the strain gage will result in a nonzero detector potential V_o. The change in this voltage can then be used to determine the corresponding strain. When the gage resistance changes, the detector voltage changes as

$$\frac{\Delta V_o}{V_e} = \frac{R_1 + \Delta R_1}{R_1 + \Delta R_1 + R_4} - \frac{R_2}{R_2 + R_3} \tag{13.4}$$

so the change in resistance of the strain gage can be expressed as

$$\frac{\Delta R_1}{R_1} = \frac{(R_4/R_1)[\Delta V_o/V_e + R_2/(R_2 + R_3)]}{1 - \Delta V_o/V_e - R_2/(R_2 + R_3)} - 1 \tag{13.5}$$

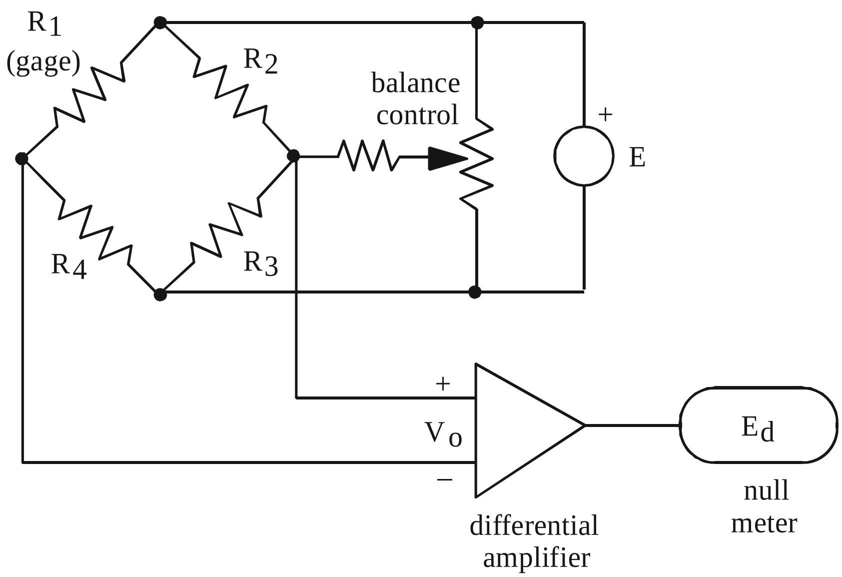


Figure 13.3 Strain Gage Conditioner Circuit

Now, turning our attention toward mechanics of materials, it can be shown that the strains ε_1, ε_2, ε_3 of the rectangular strain gage rosette are related to the principal strains and principal stresses as follows:

$$\varepsilon_a, \varepsilon_b = \frac{\varepsilon_1 + \varepsilon_3}{2} \pm \frac{1}{\sqrt{2}}[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2]^{1/2} \tag{13.6}$$

$$\sigma_a, \sigma_b = \frac{E(\varepsilon_1 + \varepsilon_3)}{2(1 - \nu)} \pm \frac{E}{\sqrt{2}(1 + \nu)}[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2]^{1/2} \tag{13.7}$$

and the direction of the maximum principal stress axis (σ_a axis) as measured counterclockwise from gage 1 is given by:

$$\tan 2\theta = \frac{2\varepsilon_2 - \varepsilon_1 - \varepsilon_3}{\varepsilon_1 - \varepsilon_3} \tag{13.8}$$

and since $\varepsilon_2 > 0.5\,(\varepsilon_1+\varepsilon_3)$, we use solution in the top half plane ($0 < 2\theta < 180°$).

The rectangular strain gage rosette senses the state of strain at a point on the top of the tubular cantilever beam. The stress due to bending at this point is:

$$\sigma_x = \frac{Mc}{I} \tag{13.9}$$

and the shear stress is:

$$\tau_{xy} = \frac{Tc}{J} \tag{13.10}$$

where

M = moment corresponding to applied load

T = torque corresponding to applied load

c = radius to outer surface of the tube

I = area moment of inertia of the tube

J = polar area moment of inertia of the tube

x = axial direction

The moment of inertia and polar moment of inertia for a tube are:

$$I = \frac{\pi}{64}(d_o^4 - d_i^4) \tag{13.11}$$

$$J = 2I = \frac{\pi}{32}(d_o^4 - d_i^4) \tag{13.12}$$

Since the shear stress on the principal planes is zero, and since σ_x is located at an angle

$$\phi = 45° - \theta \tag{13.13}$$

from the principal axes (from Mohr's Circle), the plane stress equations give us:

$$\sigma_x = \sigma_{avg} + \frac{\sigma_a - \sigma_b}{2}\cos 2\phi \tag{13.14}$$

$$\tau_{xy} = \frac{\sigma_a - \sigma_b}{2}\sin 2\phi \tag{13.15}$$

where $\sigma_{avg} = 0.5\,(\sigma_a + \sigma_b)$.

13.3 Laboratory Procedure / Summary Sheet

Group: ____ Names: ___________________________ ______________________________

___________________________ ______________________________

The experimental setup is illustrated in Figure 13.4. We wish to determine the bending moment M (theoretical value = mgb), the torque T (theoretical value = mga), and the mass m by utilizing the strain gage measurements given.

Properties and geometry of the aluminum tube, strain gage rosette, and hanging mass:

E = 70 GPa, $\nu = 0.334$
L = 0.395 m, a = 0.16 m, b = 0.182 m
d_o = 1.00 in, t = 0.085 in
F = 2.05
m = 1.492 kg

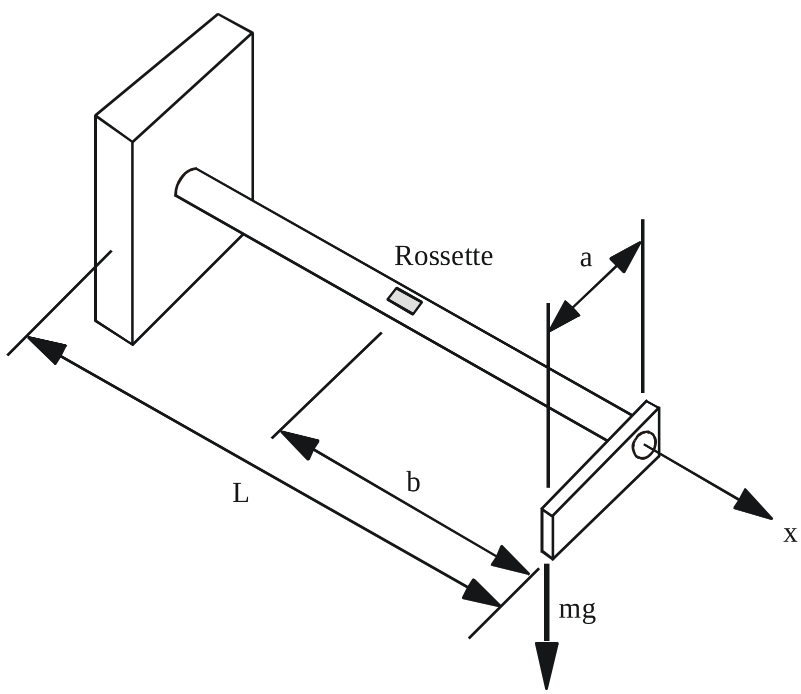


Figure 13.4 Experimental Setup

(1) It can be shown (see "Experimental Stress Analysis" by Dally and Riley, McGraw-Hill, 1991) that an active gage (with gage factor F) produces approximately F/4 output microvolts per microstrain and per volt excitation. NOTE - this is a unitless quantity: $\frac{\mu V}{\mu\varepsilon V}$. Thus the equation relating strain to measured voltage is:

$$\varepsilon = \frac{V_{meas}/GAIN}{\left(\frac{F}{4}\right)V_{ex}} \tag{13.16}$$

For an excitation potential of 5 volts, we wish to find the gain of the amplifier required to produce a 2 volt output at 500$\mu\varepsilon$. Calculate the required gain assuming a gage factor of 2, realizing that the gage factors for the gages being used might be different.

Gain = _______________

(2) The strain gage rosette is connected to channels 1, 2 and 3 on the 2120A. Make sure that the gain multiplier control is set to x200. Now set the gain control dial based on the value calculated in part 1; i.e. set the gain control to gain/200 for channels 1 – 3.

(3) Make sure that there is no external load applied to the cantilever. Now adjust the bridge balance for each channel (1 – 3); First turn the EXCIT toggle ON and rotate the BALANCE control until both output lamps are extinguished. If the (-) lamp is illuminated turn the BALANCE control clockwise. Conversely, if the (+) lamp is illuminated turn the BALANCE control counterclockwise. If you are having difficulty distinguishing whether or not the lamps are illuminated, you may use a voltmeter attached to the DAC interface card to zero the bridge potential. Under no load conditions each channel should read zero volts; adjust the BALANCE control accordingly.

(4) Hang the mass from the center of the tube and record the gage voltages. Comment on these results.

(5) Hang the mass at the end of the lever arm. Using a voltmeter, read the voltages corresponding to gages 1, 2 and 3, and record them below. Also, be sure to measure the actual excitation voltage on each bridge using the selector knob and ports on the right side of the bridge unit.

V_1 = _______________ V_{1ext} = _______________

V_2 = _______________ V_{2ext} = _______________

V_3 = _______________ V_{3ext} = _______________

(6) Now calculate the strains in each of the three gages of the rosette; utilize the relationship from part 1.

ε_1 = ______________

ε_2 = ______________

ε_3 = ______________

(7) Knowing these strains determine the following:

a) The bending moment in the beam associated with the applied load.

b) The torque produced by the lever arm and the applied load.

c) The mass applied at the end of the lever arm.

(8) Submit your full analysis used to determine the value of the hung mass from the strain gage voltage measurements. Compare the calculated result to the actual value of the mass. Submit your work to your TA at the following week's Lab meeting. Comment on various possible sources for error in the measurements and analyses.